

Risk-Based Asset Allocation: *A New Answer to an Old Question?*

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The global financial crisis in 2008 caused investors to question what went wrong with many of their portfolios, which were believed to be diversified. Mean-variance optimization (MVO), 60/40, modern portfolio theory (MPT), and others seem to have been put on trial by practitioners and critics alike for their apparent underdiversification and accused failure to provide risk control.¹ A list of “new paradigms” or “next generation solutions” has been declared to displace MPT.² A growing amount of literature on portfolio construction approaches focused on risks and diversification rather than on estimating expected returns, collectively called risk-based asset allocation in this study, has been documented.

On the topic of strategic asset allocation, we have been seeing more writings on the various versions of risk-based approaches applied to a global universe of assets, especially in cases of pension and endowment management. Allen [2010] and Foresti and Rush [2010] provide good examples. A common finding among these studies is the superior risk-adjusted return of a portfolio that is constructed in such a way that assets are expected to contribute equal risk to the whole portfolio—an approach commonly labeled risk parity. In a risk parity approach, only risk forecasts are used as inputs, and no forecasts of returns of any assets are required.

In recent years, we have also witnessed a growing literature documenting, in particular, that some equity portfolios—naively diversified portfolios (e.g., equally weighted portfolios), portfolios that are constructed to achieve the minimum volatility possible given the universe of risky assets (e.g., stocks), in order to achieve “maximum diversification” subject to the definition of diversification, or portfolios that are dubbed as risk parity—outperformed on a risk-adjusted basis both the market capitalization-weighted portfolio and portfolios that ex ante are constructed to be mean-variance optimal as derived by application of the Markowitz optimization. In some studies, these portfolios even outperformed the market portfolio on an absolute return basis. Examples include Clarke, de Silva, and Thorley [2006]; DeMiguel, Garlappi, and Uppal [2009]; Behr, Güttler, and Miebs [2008]; Martellini [2008]; and Choueifaty and Coignard [2008]. We have also seen the parallel development in the industry of the growth of product offerings and client interest in investment vehicles built upon these findings, especially in equities (Johnson [2008]). One common characteristic across all of these portfolios is that the only input required to determine the portfolio compositions is a model of risk, which is typically measured by the covariance matrix, while explicit modeling of expected returns is not required. Some of these studies argue

that the superior performance of these portfolios is the result of better diversification.

Why do these seemingly return-insensitive portfolios outperform both the market capitalization-weighted portfolios as well as those that make an explicit effort to predict returns and are optimized ex ante to be mean-variance efficient? While studies such as those by Lindberg [2009] and Maillard, Roncalli, and Teiletche [2010] shed some light on understanding the properties of these risk-based portfolios, to date we have not identified one theory that *predicts*, ex ante, that any of these risk-based portfolios should be more efficient than other portfolios.³ If such a theory indeed exists, it would represent a profound finding—investors who are ignorant of returns are predicted to outperform investors who make an effort to predict returns. It would also have interesting implications for the state of the market's informational efficiency; in such a world, investors would stop seeking valuable information on asset prices, yet still would expect to perform well. In the context of the Grossman and Stiglitz [1980] paradox, the information content of asset prices could become obscure.⁴

In this article, we begin with a brief review of the underlying economic meanings of mean-variance efficiency. As these underpinnings of investment theories are questioned, we believe that putting them into context can help frame the current debate. We then discuss the conceptual underpinnings and characteristics of some published risk-based approaches to determine asset weights. We next discuss portfolio return due to diversification—a value used by many proponents of risk-based asset allocation approaches to explain their outperformance. Rather than simulating the historical performance of these approaches as case studies—performance that is dependent upon sample, universe, and time period, among others—we compare and contrast these asset allocation approaches by constructing a snapshot of a U.S. 10-sector portfolio. While we by no means draw any conclusions based on just one example in a particular universe, when studying the risk characteristics of these resulting portfolios, we found that, for example, the portfolios that are interpreted as being “most diversified” based on one definition of diversification can have among the most concentrated weights and risk contribution profiles. While the application of risk-based approaches can be applied to any given investment universe, we have chosen equities for the ease of

illustration, since market values of publicly traded equities can easily be determined.

Using this oversimplified world of risky assets, we attempt to show that the underperformance of the market capitalization-weighted portfolio relative to some alternative asset allocation approaches should not be surprising. The key is that we do not know, ex ante, which portfolio will outperform the market. Additionally, we agree that 1) any portfolio that deviates from the market portfolio is active irrespective of its construction methodology, and 2) portfolios that *consistently* outperform the market must have better knowledge of the return characteristics of the asset's universe than the market. We argue that risk-based portfolios, as well as others, are no exception.

Although we use the equity universe for the sake of illustrating our points, our discussions and conclusions are also applicable when considering the asset allocation of a pension plan that includes multiple global asset classes. While some assets within these plans do not have well-defined concepts of market value—such as hedge funds, private equity, and commodities, among others—the market portfolio concept is still valid. For instance, as a rough starting point, the unobservable market portfolio may be approximated by aggregating the portfolio holdings of the biggest pension plans in the world. We conclude by sharing our thoughts on whether risk-based asset allocation is a new answer to an old question.

But what exactly is the “old question” being asked? Asking what the old question is truly about reveals a common challenge for many of these risk-based approaches—they lack a clear statement of their objectives. Why do investors want a minimum-variance portfolio? Before we begin to discuss how to achieve maximum diversification, we should ask investors why they wish to achieve it in the first place. What exactly does a risk parity portfolio try to achieve? For example, in order to evaluate the performance of a minimum-variance portfolio, the fair metric should be the comparison of the realized volatility of such a portfolio with that of the lowest among other portfolios ex post. To evaluate the portfolio labeled as the most diversified portfolio, one should investigate if that portfolio, constructed accordingly, is indeed the most diversified. By the same token, to evaluate the risk parity portfolio, one should study, ex post, whether the risk contributions from assets are indeed equal, as they are constructed

to be. Because construction of risk-based portfolios is insensitive to expected excess returns as inputs and is entirely risk based,⁵ fair evaluations of these portfolios should therefore also be entirely risk based.

Interestingly, all of the studies evaluate the performance of their proposed risk-based portfolios in the context of risk-adjusted return measures such as the Sharpe ratio, as if returns were built into the objective functions dictating how these portfolios are to be constructed. The objective function (or lack thereof), *ex ante*, and the performance evaluation, *ex post*, are inconsistent. Since performance goals, such as maximizing the Sharpe ratio, are not incorporated into their objective function, some of these risk-based portfolios cannot be uniquely determined and remain heuristic in nature. Without a clearly defined objective, we struggle to understand what problems these portfolios are built to solve.

While proponents of these portfolios may not construct them, nor explicitly claim them to be mean-variance efficient, there is no doubt that their performance metrics reveal their preference for more portfolio efficiency than less. At the very least, one may interpret that these risk-based approaches were built to approximate a mean-variance efficient world, for various reasons such as to mitigate the well-known problem of error maximization or other problems yet to be clarified. Until each of these portfolios clearly defines its investment objective, we suggest that mean-variance efficiency remains the best metric of evaluation, although we acknowledge that it can be unfair.

By now, we can restate the old question as: How do we improve portfolio efficiency? Studying the conditions under which these risk-based portfolios will be mean-variance efficient can help us understand the type of investment world they attempt to approximate.

MEAN-VARIANCE EFFICIENCY

MVO portfolios are designed to deliver the highest returns given the levels of risk, where risk is typically measured by the standard deviation or volatility of returns. As long as input parameters such as forecasts of returns, risks, and risk aversion are defined, MVO portfolios can be uniquely determined.⁶ The tangency portfolio of risky assets is a special MVO portfolio whose Sharpe ratio is maximized; we label this portfolio as the MSR portfolio. In targeting risk while maintaining efficiency, the fund separation theorem of Tobin [1958]

and Merton [1973] suggests that one should hold a combination of the risk-free asset and the MSR portfolio, but nothing else.

In economics, it is well known that optimality of profit maximization is achieved when marginal revenue is equal to marginal cost. When constraints are ignored, as it has been established by Scherer [2007]⁷, there is a very special meaning behind the MSR portfolio, which is closely tied to the concept of profit maximization in economics. In the case of financial investments, marginal revenue is equivalent to returns in excess of cash at the marginal cost, which is measured by risk. The MSR portfolio is a portfolio in which any small changes to the asset allocation decision for the set of current portfolio weights will lead to a deterioration of efficiency. When this state is achieved, the portfolio is said to be the MSR portfolio. In equation form, the MSR portfolio is

$$\begin{aligned} & \frac{\text{Exp Excess Return of Asset } i}{MCTR \text{ of Asset } i} \\ &= \frac{\text{Exp Excess Return of Asset } j}{MCTR \text{ of Asset } j} \\ &= \text{Portfolio's Sharpe Ratio} \end{aligned} \quad (1)$$

where MCTR stands for marginal contribution to risk. It is obvious that the MSR portfolio is jointly determined by the results of evaluating both expected excess returns and risks of all assets in the universe. As a result, the risk contribution of assets is determined to be optimal, given the return and risk forecasts as inputs.

Proponents of “new generation solutions” often claim better diversification as the reason behind the apparently better performance of their approaches. As Meucci [2009] pointed out, “[t]here exists no broadly accepted, unique, satisfactory methodology to precisely quantify and manage diversification” (p. 74). Subject to the return and risk forecasts as inputs to MVO, MPT argues that, in fact, the MVO portfolio is engineered to achieve the best diversification possible, given the universe of assets. Today, statements are often made that a 60/40 portfolio in stocks and bonds as determined by mean-variance optimization is not well-diversified because about 90% of risks come from stocks. Although such a statement may be valid, we would point out that if the 60/40 portfolio is determined to be the most efficient, subject to the inputs and any constraints⁸ to the optimization, the resulting risk attribution is simply a

reflection of the inputs. As long as the investor optimizes, there are reasons behind each portfolio being deemed efficient ex ante, and one should not judge a portfolio as poorly diversified solely based on its resulting risk attribution analysis without first cross-checking against the inputs. If the resulting risk attribution of any ex ante efficient portfolios indeed causes concerns, the investors may revisit the inputs and revise them in such a way that portfolio optimization can become an iterative process.

RISK-BASED PORTFOLIOS

In this section, we discuss the characteristics of four published risk-based portfolio construction methodologies. In the appendix, we provide more mathematical descriptions about how these portfolios are constructed, and we also discuss a relatively newly published approach that we find interesting.

Case 1: Equally Weighted Portfolio

The equally weighted (EW) portfolio is probably the simplest portfolio construction approach in an attempt to achieve diversification. In an EW portfolio, all assets are given the same weight. That is, for n assets, each asset will be assigned a weight equal to $1/n$. This portfolio completely ignores which assets are being invested. Technically this means that there is no objective function associated with the EW portfolio, yet it can be uniquely determined. Since it completely ignores the return and risk prospects of investments, one may argue that it is the most heuristic portfolio.

If all assets have the same correlations with each other, as well as identical returns and volatilities, it can be shown that the EW portfolio is indeed an MVO portfolio. In their extensive study of 14 different asset allocation models applied to seven different datasets of the global equities universe, DeMiguel, Garlappi, and Uppal [2009] concluded that none of these theoretically sound asset allocation approaches are consistently better out of sample than the heuristic $1/n$ equally weighted rule, and their results suggest that “[t]here are still many ‘miles to go’ before the gains promised by optimal portfolio choice can actually be realized out of sample” (p. 1915). The authors, however, also emphasized that by no means do they advocate using $1/n$ as an asset allocation strategy, but rather as a benchmark to assess the performance of

other portfolio allocation rules. In a more recent study, however, Kritzman, Page, and Turkington [2010] argued that with some naive inputs not reliant on rolling short-term samples for estimating expected returns, optimized portfolios usually outperform EW portfolios.

Does equal weighting guarantee diversification? It depends. Because such a strategy completely ignores the characteristics of assets, the nature of the resulting portfolio, such as the degree of diversification, can become highly sensitive to the universe of assets under consideration. For example, in the Russell 1000 universe of equities, the consumer discretionary and consumer staples sectors were both weighted at approximately 11% as of March 2010. An equally weighted portfolio of all stocks in the Russell 1000 universe, however, would put the weight of the consumer discretionary sector at almost three times the weight of the consumer staples sector due to the difference in the number of exchange-traded stocks grouped in each of these two sectors. The degree of diversification in this portfolio is important, but the more interesting question is what makes an investor allocate more to stocks grouped under a particular cluster, such as a sector, simply because these stocks outnumber the other clusters? Also, if the risks of these assets are very different, equal weighting will still lead to risk concentrations as the EW portfolio has the same weight on the highest-risk asset and the lowest-risk asset. Clearly, all else being equal, the EW portfolio prefers assets with lower market value.

Lastly, the extent to which the $1/n$ strategy can be implemented also depends on the particular universe of assets. Dash and Loggie [2008] argued that the turnover and capacity constraints of equally weighted indices relative to market capitalization-weighted indices are overstated in the S&P 500 universe. As this strategy requires constant rebalancing after price movements, management of turnover and transaction costs can become critical in other universes that include less-liquid or smaller-capitalization securities.

Case 2: Global Minimum-Variance Portfolio

The global minimum-variance (GMV) portfolio plays a unique role in a portfolio frontier. Ex ante, the GMV portfolio is the portfolio of risky assets that is expected to have the lowest possible volatility and that can be uniquely determined merely by a covariance

matrix. It is also the only portfolio that is on the efficient frontier without expected returns as inputs.

In the Clark, de Silva, and Thorley [2006] study of the 1,000 largest-capitalization stocks in the U.S. from 1968 to 2005, various versions of the GMV portfolio are found to have higher returns and lower volatilities than the market portfolio *ex post*. In another study, Behr, Güttler, and Miebs [2008] reported that with the entire Center for Research in Security Prices (CRSP) dataset from April 1964 to December 2007, many different GMV portfolios with different constraints on weights outperformed the market capitalization-weighted index with higher realized returns, lower realized volatility, and therefore better risk-adjusted performance. In addition, the high sensitivity of the GMV portfolios to the frequency of rebalancing and the imposed weight constraints are also noted. The results are interesting and profound; the very special efficient portfolio—the GMV in these cases—that is engineered *ex ante* to deliver the lowest volatility and lowest return along the efficient frontier turns out to have had a higher return than the market portfolio.

Is the GMV portfolio diversified? Other than being the only *ex ante* efficient portfolio completely insensitive to expected returns, matrix algebra reveals that it is also the only portfolio in which the marginal contribution to risk of all assets is identical. This by no means suggests that risk contributions of assets are equal, as assets can have different volatilities and different weights in the GMV portfolio. It has also been documented that the GMV portfolio tends to be a concentrated one, since it must load up on assets that have low volatilities. In the appendix, we show that the percentage contribution to risk of each asset is equal to the portfolio weight of the same asset in the GMV portfolio. Therefore, the fact that the GMV portfolio is expected to have the lowest volatility, given the universe of risky assets, does not necessarily suggest that it is diversified from the standpoint of portfolio weights and risk contributions. Because of its tendency to have high weights on low-volatility assets, its asset weights can thus be more sensitive to estimates of both volatilities and correlations. Recognizing these characteristics, studies such as those of Clark, de Silva, and Thorley [2006] typically apply some constraints to limit potential weight concentration.

Finally, why would an investor want the GMV portfolio? Could it be because the investor prefers lower volatility? If that is the reason, the MPT suggests that

a better way to achieve lower volatility and better risk-adjusted return is through a combination of cash and a portfolio that is more efficient than the GMV; this approach is known as the fund separation theorem (Tobin [1958] and Merton [1973]). The theory goes further, stating that the only portfolio of risky assets an investor should hold is the MSR portfolio in which combinations with cash will achieve different preferred risk levels, but all combinations will have the maximum degree of portfolio efficiency. Therefore, from a theoretical standpoint, perhaps the rationale for an investor to prefer the *ex ante* GMV portfolio would be the expectation that it will be the more efficient portfolio *ex post*, which is itself an active investment view. In the appendix, we also establish the result that when expected excess returns of all assets are identical, the GMV is optimal.

Case 3: Most Diversified Portfolio

Choueifaty and Coignard [2008] illustrated how to achieve what they interpret as “maximum diversification” within a universe of assets. The authors first introduced the measure known as the diversification ratio, which is a ratio of the weighted average of the volatilities of assets to the volatility of the portfolio of the same assets. The authors’ interpretation is that the higher the ratio, the more diversified the portfolio is. The most diversified portfolio (MDP), given a universe of assets, is the portfolio with weights of assets that maximize the diversification ratio. The authors report that during the sample period December 1990–February 2008, the MDPs of the S&P 500 universe and the Dow Jones Euro Stoxx Large Cap universe significantly outperformed the market capitalization-weighted portfolios as well as the corresponding GMV and EW portfolios, delivering higher returns and lower volatilities. Note that in addition to various constraints on weights and sum of weights similar to the study of GMV, constraints on contribution to risk per asset are also imposed in constructing the MDP in their study; these will be discussed in later sections.

While the construction of the MDP is straightforward, whether it is indeed a diversified portfolio is subject to interpretation and different definitions of diversification. It should be noted that the MDP is constructed to maximize the distance between two volatility measures of the same portfolio, namely, the volatility of the portfolio in an imaginary state in which there is absolutely no

diversification, and the volatility of the same portfolio in the real world where there is indeed some diversification. Meucci [2009] correctly pointed out that the difference between the weighted sum of the volatilities of each position and the total portfolio volatility is a *differential* diversification measure, not an absolute measure of how diversified the portfolio is.

Others evaluate the degree of diversification of a portfolio on an absolute basis in the realistic investment world, rather than on a relative basis as advocated by the design of the MDP; for example, Levell [2010, p. 1] considered diversification maximized when taking equal risk in each investment. In other words, it cannot be ruled out that the MDP that maximizes the differential, relative diversification measure known as the diversification ratio can itself be a relatively concentrated portfolio that is not as diversified when judged by other definitions of diversification. For instance, in their example of the Eurozone benchmark, Chouiefaty and Coignard [2008] drew the conclusion that the MDP is always about “1.5 times as diversified as the benchmark” (p. 46), based entirely on the observed diversification ratios of the two portfolios rather than based on how a typical investor might evaluate if a portfolio is diversified. We later demonstrate with an example that, subject to our estimates of the covariance matrix, a U.S. sector MDP is a relatively concentrated portfolio both on portfolio weights and risk contributions, and therefore is likely to be judged not as diversified as the, for instance, market capitalization-weighted portfolio, even though the MDP maximizes the diversification ratio by design.

While the benefits of diversification are made clear in MPT and irrespective of the fact that no broadly accepted way of defining and managing diversification exists (Meucci [2009]), we struggle to understand why investors should *maximize* diversification. Therefore, without a clear investment objective function behind maximizing the diversification ratio, it is unclear what investment problem the MDP is built to solve. Chouiefaty and Coignard [2008] stated that their objective was to “investigate the theoretical and empirical properties of diversification as a criterion in portfolio construction” (p. 40). Specifically on “empirical properties,” the authors reported statistics including return, excess return, volatility, and the Sharpe ratio, together with the diversification ratio as defined, and results from standard style analysis. Following the metrics used by the authors, we interpret the MDP as an attempt to approximate

and/or improve mean-variance efficiency, although we acknowledge that its true underlying objective, yet to be stated, can be different.

In the appendix, we show that derivation of the MDP can be substantially simplified when viewed from within the context of mean-variance efficiency. When analyzed with the objective of achieving mean-variance efficiency, the MDP is an active portfolio in which the expected excess returns of *all* assets are modeled as the same multiples of their expected volatilities, a point also correctly identified by Chouiefaty and Coignard [2008, p. 41]).⁹ Stated differently, identical Sharpe ratios for all assets is a necessary condition for the MDP to be an efficient portfolio in a universe of all assets. This brings us into the heart of the debate. Suppose X and Y are two assets in the universe with identical Sharpe ratios. A new company, Z, can be created by holding shares of X and Y on the balance sheet. Z is now traded and becomes a new asset in the universe. The Sharpe ratio of Z is higher than the Sharpe ratio of X and Y unless the correlation between X and Y is +1, which means X and Y are redundant. If the correlation is anything but +1, then the higher Sharpe ratio of Z violates the necessary condition of identical Sharpe ratios, and an arbitrage opportunity exists.

By definition, a risk-based approach does not require explicit estimates of expected returns. We note that any argument applied to a subset of assets, rather than to the universe of all assets, would require grouping the assets based on different Sharpe ratios, a step violating the spirit of risk-based asset allocation. Therefore, our previous arguments emphasize identical Sharpe ratios of *all* assets instead of just a subset of assets. Regardless of what the true underlying investment objective behind MDP is, be it the same or different from mean-variance efficiency, it is significant to note that an investor with the view of identical Sharpe ratios for all assets is shown to hold the MDP in an attempt to achieve mean-variance efficiency. In the context of mean-variance efficiency, the MDP may be interpreted as an attempt to approximate a world in which all the assets have identical Sharpe ratios but yet have correlations that differ from +1, which implies that an arbitrage opportunity exists.

Case 4: Risk Contribution Portfolio

To begin, the risk contribution of an asset is defined as the simple product of its weight in the portfolio and its

marginal contribution to risk; the details can be found in the appendix. Risk contribution (RC) portfolios generally refer to portfolios that are constructed to achieve a predetermined profile of risk contributions by asset. Anecdotal observations seem to suggest that the special case known as the risk parity (RP) approach—with equal risk contribution from each asset in the portfolio—is gaining the most traction. Studies such as those by Allen [2010], Foresti and Rush [2010], Levell [2010], and Maillard, Roncalli, and Teiletche [2010] helped shed more light on this approach. As a result, we will use the RP as a special case in discussing the properties of RC portfolios.

In the appendix, we develop the result that RP portfolio weights are inversely proportional to the portfolio's betas with respect to the assets; in other words, the higher the volatility and/or the correlation of an asset with other assets, the lower its weight in the RP portfolio. Unlike other risk-based approaches, the RP approach does not have an analytical solution because portfolio weights are endogenous in determining the risk contribution of an asset in the portfolio. Although it has been proved by Maillard, Roncalli, and Teiletche [2010] that the solution for the RP approach always exists, finding this solution numerically can quickly become tricky because the number of possible sets of portfolio weights grows exponentially as the number of assets in the universe increases. As a result, in practice, the RP approach also requires arguably subjective investor views to first pre-group assets into a manageable number of component portfolios. Next, weights of these portfolios can be determined in such a way that each portfolio is expected to contribute equal risk; this approach is dubbed risk parity. Unlike in MVO where we are guaranteed to have an optimal solution due to the convex nature of the problem, there is no such guarantee that the RP portfolio solution we obtain numerically achieves global optimality, and the possibility of multiple numerical solutions cannot be ruled out entirely. The bottom line is that achieving the risk parity condition remains somewhat heuristic in nature.

For example, an RP portfolio of a universe with 500 stocks requires finding a set of portfolio weights so that the risk contribution from each stock is identical, a numerical optimization problem that is more challenging to solve.¹⁰ Instead, one may first group stocks into, for example, 10 sectors so that each sector is expected to contribute equal risk in the RP portfolio. However, the

methodology to weight individual stocks within each sector—such as market capitalization-weighting, equal weighting, and others—has to be first determined, and this step is often heuristic. Of course, different weightings of individual stocks within sectors would lead to different sector weights in the RP portfolio. Moreover, grouping by sectors is only one of many possible ways to group stocks into a manageable number of assets, not to mention that sector classification on its own is often debatable. For the sake of debate, if one views a company (a portfolio) as a collection of several risky lines of business (assets), do these lines of business within the company even conform to the spirit of risk parity? Some subjective elements seem to be unavoidable. Perhaps it is this additional degree of freedom that makes investors feel more in control with this approach, and therefore deem it to be a more sensible portfolio as a result, provided that investment experience and knowledge can be integrated into the heuristics.

The traditional MVO approach uses expected excess returns and risk forecasts as inputs, while weights and the resulting risk contributions of the optimal portfolio are the outputs. Note that in the mean-variance paradigm, given the portfolio weights, risk contributions simply reflect the inputs to the optimization. If the investor is uncomfortable with the risk contribution profile, what the investor is truly uncomfortable with is the set of inputs. Thus, one may view the RC approach that uses targeted risk contributions as guidance in constructing portfolios as potentially a complimentary way to help investors refine their forecasts of joint return characteristics. A distinct advantage of these RC portfolios is that the investor will more likely find the resulting portfolio reasonable and intuitive because the targeted risk contributions were selected by the investor as the starting point in the portfolio construction process. To that end, the parity element of the RP approach can be of secondary importance to some investors who adopt the RC approach. Instead, it is the ability to specify a preferred risk contribution profile that makes the RC approach unique and attractive. In addition, Qian [2006] illustrated that risk contributions as computed do have a financial interpretation that is linked to quantifiable economic losses.

Maillard, Roncalli, and Teiletche [2010] showed that the necessary conditions for the RP portfolio to be efficient require identical Sharpe ratios and identical correlations among all assets in the universe. As discussed

in an earlier section, such conditions imply that all assets in the world are redundant. In the context of mean-variance efficiency, the RP portfolio may be interpreted as an attempt to approximate a world in which there is only one risky asset. Therefore, the efficiency of the RP portfolio does not violate the no-arbitrage condition and the risk parity condition always holds because all assets are statistically the same in such a world.

RETURN DUE TO DIVERSIFICATION

Some studies of risk-based portfolios suggest that better diversification is a reason for the outperformance of these portfolios, often citing Booth and Fama [1992], who attempted to quantify the additional return that an asset can be expected to contribute to the compound return of a portfolio as a result of diversification. In this section, we take a more detailed look at the potential benefits of diversification.

As illustrated in Equation 2 of Booth and Fama [1992], the compound return of asset j can be expressed as

$$C_j = \ln[1 + E(R_j)] - \frac{s_j^2}{2\ln[1 + E(R_j)]^2} \quad (2)$$

where $E(R)$ denotes expected return, and s denotes standard deviation. When this same asset j is put into a portfolio with other assets, Booth and Fama [1992] showed, using their Equation 5, that its contribution to the compound return of portfolio p becomes

$$C_j = \ln[1 + E(R_j)] - \frac{b_{jp} s_p^2}{2\ln[1 + E(R_j)]^2} \quad (3)$$

where b denotes the beta of one asset with respect to the portfolio. That is, the so-called return due to diversification of an asset is equal to the difference of the two terms above, given as

$$\text{Return due to diversification} = \frac{s_j^2 - b_{jp} s_p^2}{2\ln[1 + E(R_j)]^2} \quad (4)$$

This component is shown to be equal to the difference between the variance of the asset and its covariance with the portfolio, scaled by a squared term of the asset's expected return. With reasonable parameters, such as

those examples given by Booth and Fama [1992], the diversification benefit is no more than one-tenth of the return of some assets and is quite insignificant in others. Therefore, while diversification certainly helps, its benefits can be small when compared with an optimization process that takes into account the differences in returns across the universe of assets.

IMPLEMENTATION: U.S. 10-SECTOR PORTFOLIO EXAMPLE

With the exception of the EW portfolio, construction of all risk-based portfolios discussed so far requires a covariance matrix as the risk model. In the literature, different ways of estimating the covariance matrix have been applied. Choueifaty and Coignard [2008] used 250 days of daily returns to estimate the covariance matrices. For the S&P 500 Index universe, there are 500 estimates of volatilities and $500 \times (500 - 1)/2$ correlation estimates, for a total of 125,250 estimates. The 250 days of return data are insufficient to produce consistent estimates of the full covariance matrix. The resulting covariance matrix is singular and non-invertible.

To avoid this problem, other studies impose factor structure in estimating the covariance matrix such that the dimension of the matrix is substantially reduced to a manageable degree. Clarke, de Silva, and Thorley [2006], for example, used both an asymptotic principal components procedure and a Bayesian shrinkage technique to avoid the problem of non-invertibility of the sample covariance matrix. The weight constraints on securities are much less binding when the Bayesian shrinkage technique is used. Scowcroft and Sefton [2006, p. 8] gave a more detailed discussion of this well-known covariance matrix invertibility problem and solutions. Perhaps the sensitivity of the resulting portfolios to the choice of the covariance matrix reflects that, again, all these risk-based portfolios are active portfolios as determined by the perception of riskiness of the assets.

It is beyond the scope of this study to evaluate different techniques of estimating the covariance matrix. To this end, we focus on a simple portfolio of 10 sectors in the Russell 1000 universe because of its manageable dimension. For simplicity, we estimate a covariance matrix of the 10 GICS sectors based on 10 years of monthly returns as of March 31, 2010. With 10 sectors, we estimate 10 volatilities and 45 correlation measures. Therefore, given our 120 months of data, the insufficient

observations problem in estimating the covariance matrix that we discussed earlier does not apply in our example. Besides, our results can easily be replicated. Within each sector, stocks are weighted by their market capitalization, a subjective decision within the context of risk parity as we discussed earlier.

Five different portfolios of the 10 sectors are examined. These include the market capitalization-weighted (Mkt-Cap) portfolio as the benchmark, equally weighted (EW) portfolio, global minimum-variance (GMV) portfolio, most diversified portfolio (MDP), and finally, the risk parity (RP) portfolio in which the risk contribution from each sector is expected to be identical. Unlike the published studies that impose multiple constraints in constructing the GMV and the MDP, the only constraint

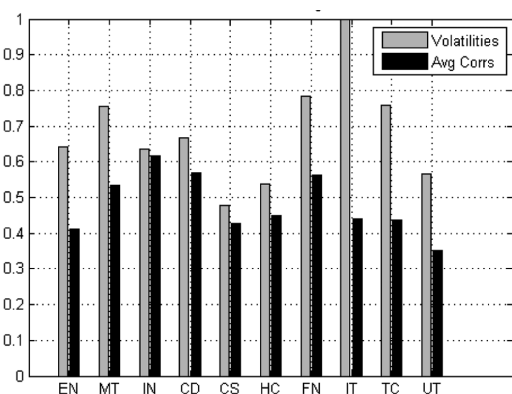
we impose is that all weights must be nonnegative. We believe that the characteristics of these portfolios constructed with minimal constraints will better expose for discussion the original ideas behind these portfolios, rather than portfolios constructed with constraints which would distort the original message.

We first examine the characteristics of the RP portfolio in Exhibit 1. In Panel A of Exhibit 1, for ease of illustration, we plot the scaled estimated volatility of each sector, scaled by the volatility of the information technology (IT) sector, which is about 30%, and the average correlation of each sector with the others. Clearly, the consumer staples sector is the least volatile sector and the utilities sector has the lowest average correlation with other sectors, justifying their higher

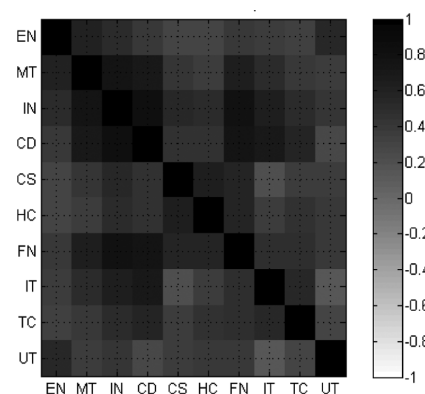
EXHIBIT 1

Characteristics of the Risk-Parity Portfolio Across GICS Sectors as of March 31, 2010

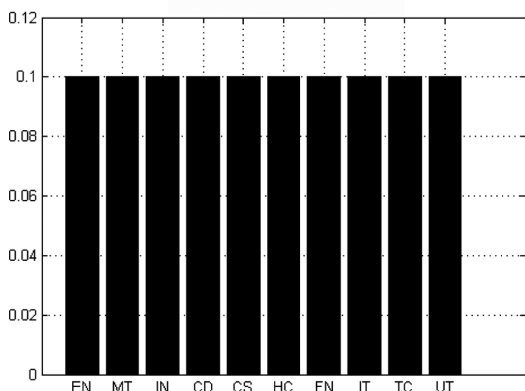
Panel A: Scaled Volatilities and Average Correlations



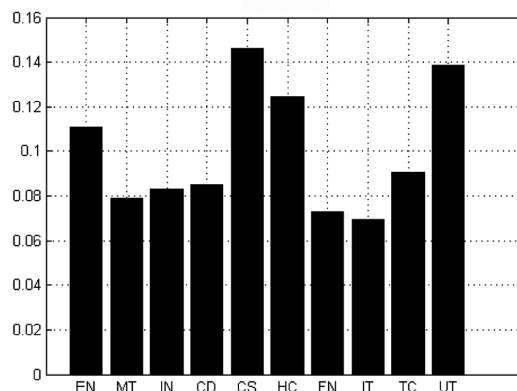
Panel B: Correlation Heat Map



Panel C: Percentage Contribution to Risk



Panel D: Weights



Note: Volatility of Information Technology = 30%.

weights in the RP portfolio. In contrast, the information technology sector has the lowest weight at 7% in the RP portfolio as a result of its highest volatility among all sectors. By construction, the percentage contribution to risk of each sector is identical at 10% as confirmed in Panel C of Exhibit 1. Panel D in Exhibit 1 reports the weights of the 10 sectors in the RP portfolio. The two sectors with the most weights are the consumer staples (CS) sector at 14.5% and the utilities (UT) sector at about 14%.

Exhibit 2 compares the weights of the five different portfolios. With respect to these weights, two of the biggest differences between the RP and the market capitalization-weighted portfolio are the weights of the financial and information technology sectors. While these two are the biggest sectors in the market portfolio by market value, they share the lowest weights in the RP portfolio as a result of their higher volatilities, and in the case of the financial sector, its higher correlations with the other sectors as well. Although the utilities sector is the second smallest in the market portfolio, it is the second largest in the RP portfolio, as discussed earlier.

The GMV portfolio has zero weight in 4 of the 10 sectors, and an almost insignificant weight in the

information technology sector. Had we not imposed the nonnegative weight constraint, it would have shorted 3 sectors.

The MDP portfolio has zero weight in 3 of 10 sectors, and only about 2% weight in the materials sector. Without the nonnegative weight constraint, it would have shorted the industrials sector. We certainly cannot overgeneralize our results based on one universe, but nevertheless our examples cast some doubt as to whether a typical investor may consider this MDP constructed from the 10 U.S. sectors a diversified portfolio.

Exhibit 3 plots the implied expected excess returns of the sectors based on the assumption that the five portfolios are mean-variance efficient.¹¹ While the weights in some sectors are very different, their implied expected excess returns are, in most cases, broadly similar in both absolute and relative magnitudes across portfolios. To some extent, these results highlight the well-known property of the MVO portfolio—its portfolio weights can be highly sensitive to inputs, particularly the return forecasts. The fact that the RP portfolio uses a targeted equal risk contribution as a starting point to search for the set of sector weights that satisfies this requirement of equal risk contribution suggests that the investor

EXHIBIT 2

Portfolio Weights Across GICS Sectors

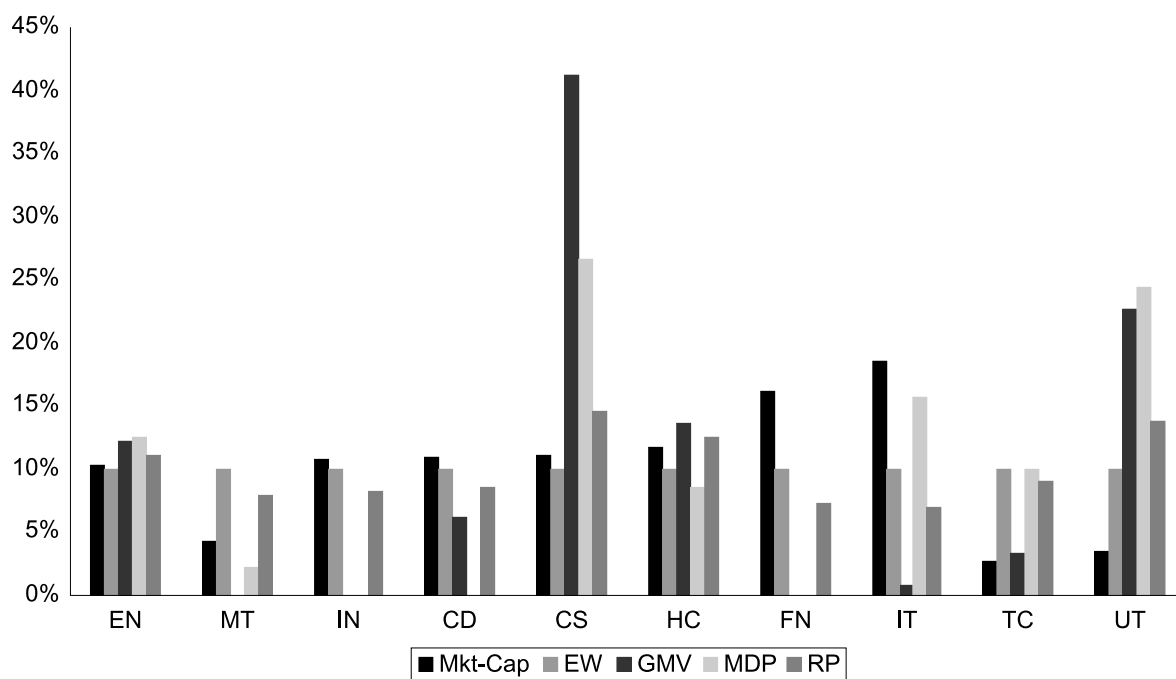
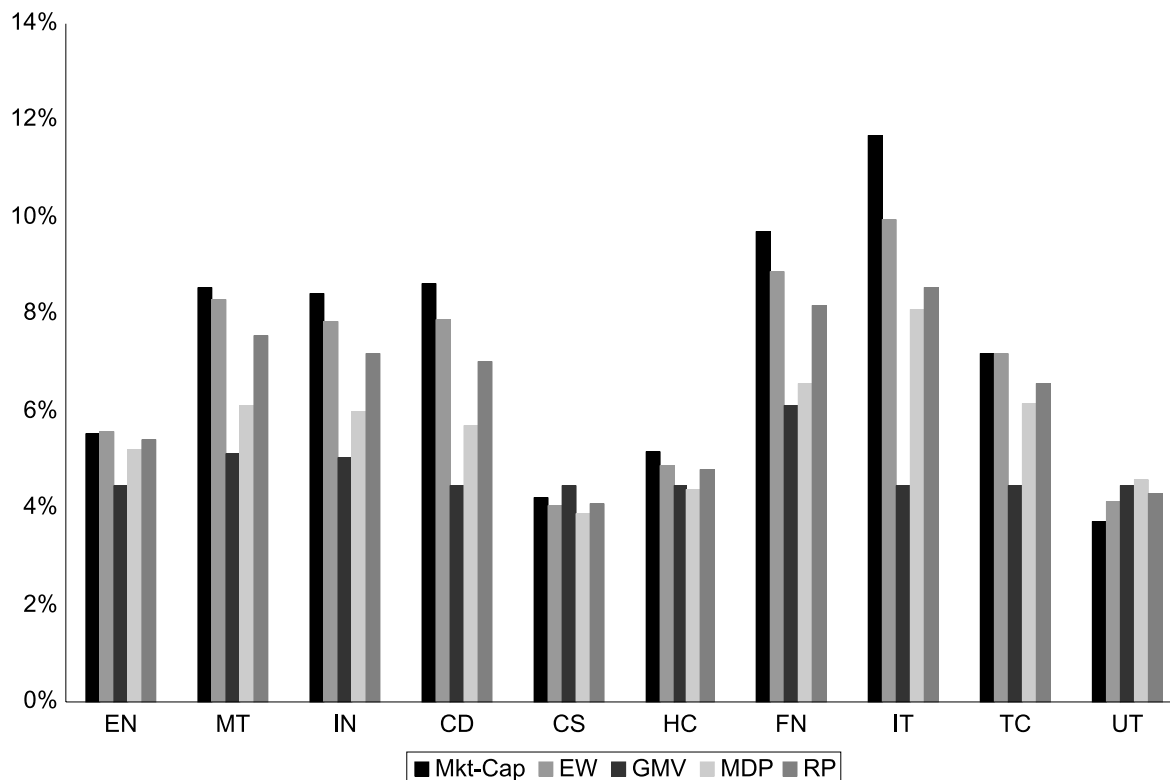


EXHIBIT 3

Portfolio Implied Expected Excess Returns for Each GICS Sector



should most likely find the sector RP portfolio acceptable, unlike in some cases of MVO where the resulting portfolio weights can be counterintuitive as documented in numerous studies. It is important to re-emphasize, however, that these targeted risk contributions embed the investor's investment views, including return and risk forecasts.

While some proponents of the alternative risk-based approaches argue that the market capitalization-weighted portfolio is not diversified, Exhibit 4 provides another perspective on the diversification of these portfolios. Although proposing a best measure of diversification is beyond the scope of this article, we believe that the risk contribution profile of a portfolio is an important set of characteristics an investor should examine when evaluating the portfolio's degree of diversification. To this end, Exhibit 4 clearly highlights the fact that the MDP and GMV portfolios, for example, are no more diversified than the market capitalization-weighted

portfolio when evaluated based on the risk contribution profile across sectors.

Lastly, Exhibit 5 plots the cumulative percentage contribution to risk of the five different portfolios. The market capitalization-weighted portfolio has a profile that sits in the middle of the five. The profile of the RP portfolio is, as expected, a straight line with each sector expected to contribute equally to risk. The EW portfolio sits between the market capitalization-weighted portfolio and the RP portfolio, while the GMV portfolio is at the top, indicating that from a risk contribution perspective, the GMV portfolio is either the most concentrated or the least diversified. Interestingly, just five sectors account for 90% of risks in the MDP, and one may argue that in this specific example the market capitalization-weighted portfolio is indeed more diversified than the MDP, which is itself among the least diversified.

EXHIBIT 4 Portfolio Sector Risk Contributions for Each GICS Sector

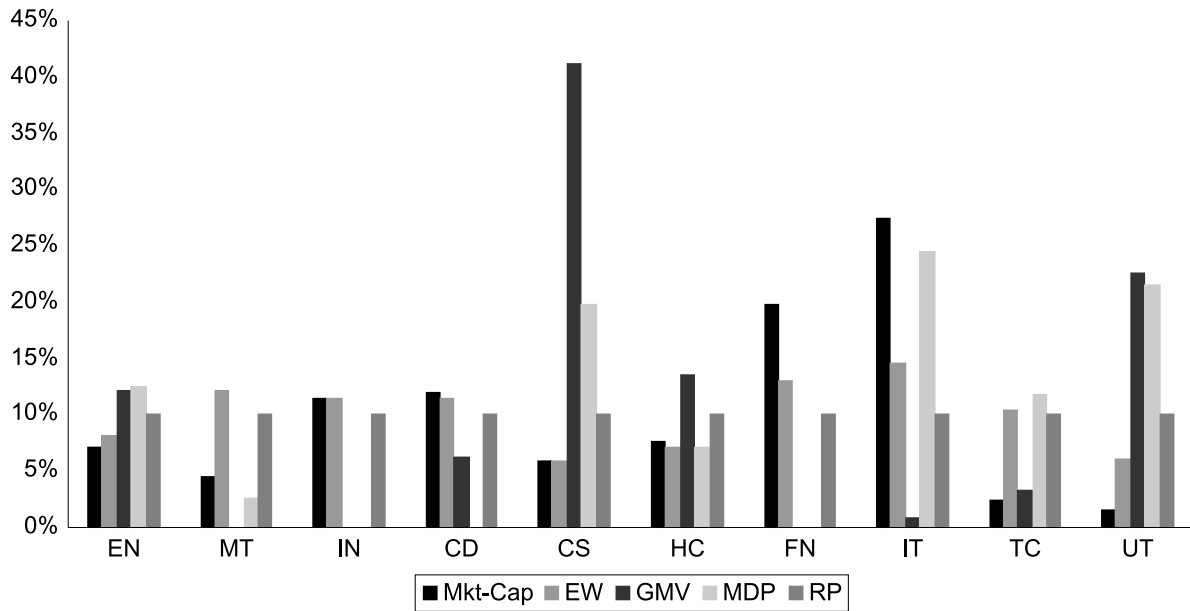
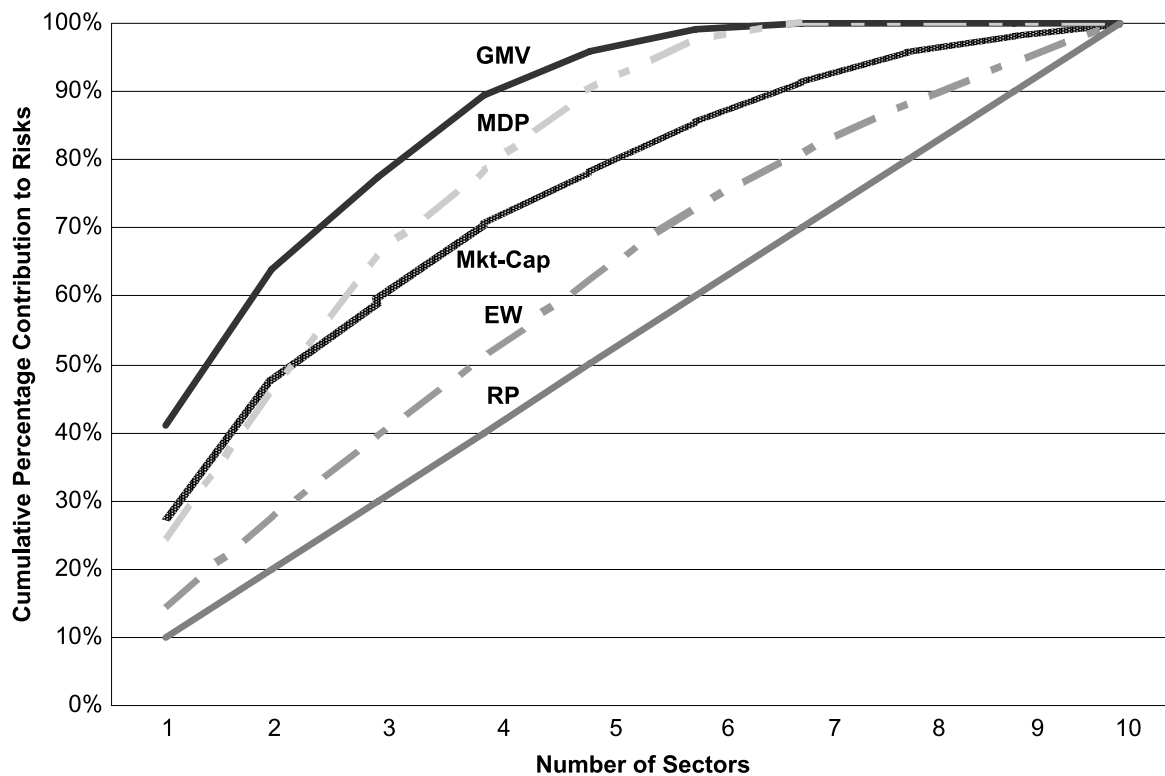


EXHIBIT 5 Portfolio Cumulative Percentage Contribution to Risk



MARKET EQUILIBRIUM, ACTIVE MANAGEMENT, AND THE EFFICIENCY OF RISK-BASED PORTFOLIOS

Perold [2007] made the following observation:

The capitalization-weighted equity market portfolio holds a special place in modern-day investing—and for good reason. The capitalization-weighted portfolio offers broad diversification and low transaction costs. Capitalization weighting is also the only strategy that all investors can follow. Because the collective holdings of investors (by definition) aggregate to the market portfolio, for every investor who is underweight a stock, another is overweight that stock and, between them, it is at best a zero-sum game. After fees and transaction costs, the average investor who deviates from capitalization weights must underperform the market portfolio (p. 31).

In one simple paragraph, Perold [2007] pinned down the very special position of the market portfolio as *the* market-clearing equilibrium, and correctly spelled out active management as “at best a zero-sum game.” In light of this, any portfolio other than the market portfolio *should* be considered an active portfolio.¹²

We must also realize that the so-called market portfolio is not constant but, instead, is changing over time. What drove the market portfolio last period to become today’s market portfolio, and what will the future market portfolio be like? A market portfolio at any point in time is a database that stores and reflects the paths of the historical realized returns of all assets in the universe through its current market capitalization asset weights. While today’s market portfolio defines the market-clearing equilibrium, it is not difficult to see that if the investment opportunity set, which broadly includes the historical joint return and risk characteristics of all assets, does not repeat itself exactly in the future, then clearly today’s market portfolio will not be the most efficient portfolio for the future.

It is true that the market portfolio is the only portfolio of risky assets that all investors *can* follow, but not every investor *has to* follow. This is where the active investment industry plays its role; there always exist opportunities to outperform the current market portfolio. Active portfolio management, in our view, is about

forecasting the future joint return characteristics of assets better than what the current market portfolio implies, which, as discussed, simply reflects realized history. In our opinion, debating the existence of outperformance against the market is not the most productive discussion. Outperformance—although a zero-sum game during a period of time—by construction always exists. Rather, the discussion should be focused on identifying other portfolios expected to outperform the market. One should not be surprised that in studies and in real life numerous portfolios have been found to be more efficient than the market portfolio during some periods of time.

To the best of our knowledge, we are not aware of any theory that predicts *ex ante* how any of the risk-based portfolios discussed in this article should perform—outperform or underperform—relative to the market. If indeed a “more diversified” portfolio is expected to outperform the market portfolio, as some proponents of risk-based portfolios seem to suggest, then it must be the case that at least one other portfolio (likely to be “less diversified” in this case) is expected to underperform the market portfolio so that the market portfolio remains the market-clearing equilibrium.

We are convinced that the capital market equilibrium concept remains our compass. The market portfolio plays a unique role in investing such that any portfolio that deviates from the market portfolio is active and outperforms the market only if it reflects more information on asset returns than the market portfolio. The collective embedded views of a portfolio that outperformed the market portfolio in a risk-adjusted sense over a period of time must have been a better set of joint return-and-risk forecasts than the market over the same period of time. Risk-based portfolios, as well as any other portfolios, regardless of how they are constructed, are no exception to this most fundamental concept of investing. As a result, we believe that in order to assess the future performance of these risk-based portfolios relative to the market portfolio, it is important to understand the conditions under which these risk-based portfolios are shown to be mean-variance optimal.

CONCLUSION

While empirical results seem to suggest that various forms of risk-based portfolios deliver better return-to-risk ratios than the market capitalization-weighted portfolio, we believe that these risk-based portfolios come

with various potential challenges, and most importantly, there is no theory to predict their performance relative to the market. As a result, we reject risk-based asset allocation as the definitive new answer to the old question of how to improve portfolio efficiency. In fact, we view risk-based approaches as a subset of the modern portfolio theory paradigm rather than as the new paradigm itself.

Of the various risk-based asset allocation approaches discussed, we consider the risk-contribution approach to be potentially more applicable because of its heuristic nature, economic intuition, and the financial interpretation that ties its concept to economic losses. We believe the flexibility to specify the preferred, targeted risk-contribution profile is an additional degree of freedom that makes the risk-contribution approach unique. The parity condition, however, is nothing more than a starting point in the absence of stronger investment views unless one truly believes that the conditions that make risk-parity optimal are prevailing.

It would be profound if an investor can be completely ignorant of future returns yet outperform the market portfolio. Any portfolio that differs from the market portfolio is by definition an active portfolio. For some active portfolios to outperform, others must underperform. Irrespective of whether expected excess returns are explicitly used as inputs, a portfolio that consistently outperforms the market must have more information on future asset returns than the market portfolio. Risk-based portfolios are no exception. They are all built to reflect our investment views, which are also known as forecasted returns and risks. Until each of the risk-based asset allocation approaches clearly states its unique investment objectives, we suggest that mean-variance efficiency remains the best metric for performance evaluation of these portfolios. To this end, modern portfolio theory remains modern.

APPENDIX

This appendix provides the technical details of how the four risk-based portfolios discussed in the article are constructed.

Definitions

N	number of assets in the universe
ω	$N \times 1$ vector of portfolio weights

ω_i	the i^{th} element of ω , denoting the weight of asset i in the portfolio
λ	risk aversion parameter
μ	$N \times 1$ vector of expected excess returns
σ	volatility of portfolio
Σ	$N \times N$ covariance matrix
Ω	$N \times N$ diagonal matrix of volatilities
V	$N \times 1$ vector of volatilities
C	$N \times N$ matrix of correlation
β	$N \times 1$ vector of betas of assets with respect to the portfolio
$\mathbf{1}$	$N \times 1$ vector of ones
I	$N \times N$ identity matrix
$MCTR$	$N \times 1$ vector of marginal contribution to risk
$PCTR$	$N \times 1$ vector of percentage contribution to risk

Mean-Variance Optimality (MVO)

It is well known that the unconstrained MVO portfolio can be determined as

$$\omega_{MVO} = \lambda \Sigma^{-1} \mu \quad (\text{A-1})$$

We can rewrite the inverse of the covariance matrix as

$$\Sigma^{-1} = \Omega^{-1} C^{-1} \Omega^{-1} \quad (\text{A-2})$$

so that the optimal portfolio in (A-1) becomes

$$\omega_{MVO} = \lambda \Omega^{-1} C^{-1} \Omega^{-1} \mu \quad (\text{A-3})$$

Risk Contribution

The marginal contribution to risk of asset i is defined as the marginal change in portfolio volatility having been given a unit change in the weight of asset i , everything else being equal. Since the variance of a portfolio is defined as

$$\sigma^2 = \omega' \Sigma \omega$$

and the vector of betas of the assets with respect to the portfolio is given by

$$\beta = \frac{\Sigma \omega}{\sigma^2}$$

the marginal contribution to risk can be derived as

$$MCTR = \frac{\partial \sigma}{\partial \omega} = \frac{\Sigma \omega}{\sigma} = \beta \sigma \quad (\text{A-4})$$

The contribution to risk of a particular asset i is the product of the weight of asset i and its marginal contribution to risk. Therefore, the percentage contribution to risk can be determined as

$$PCTR = \frac{\bar{\omega} \cdot MCTR}{\sigma} = \bar{\omega} \cdot \beta \quad (\text{A-5})$$

Case 1: Equally Weighted Portfolio (EW)

The EW portfolio is simply given by

$$\bar{\omega}_{EW} = \frac{\mathbf{1}}{N} \quad (\text{A-6})$$

The MCTR and PCTR can be determined by substituting Equation (A-6) into Equations (A-4) and (A-5) as

$$MCTR_{EW} = \frac{\Sigma \mathbf{1}}{\sqrt{\mathbf{1}' \Sigma \mathbf{1}}}$$

and

$$PCTR_{EW} = \frac{1}{N} \frac{\Sigma \mathbf{1}}{\mathbf{1}' \Sigma \mathbf{1}}$$

respectively.

Case 2: Global Minimum-Variance Portfolio (GMV)

The GMV can be derived by solving

$$\begin{aligned} \min \quad & \frac{1}{2} \bar{\omega}' \Sigma \bar{\omega} \\ \text{subject to} \quad & \bar{\omega}' \mathbf{1} = 1 \end{aligned}$$

It can be shown that

$$\bar{\omega}_{GMV} = \frac{\Sigma^{-1} \mathbf{1}}{\mathbf{1}' \Sigma^{-1} \mathbf{1}} \quad (\text{A-7})$$

The variance of the GMV portfolio is

$$\sigma_{GMV}^2 = \bar{\omega}'_{GMV} \Sigma \bar{\omega}_{GMV} = \frac{1}{\mathbf{1}' \Sigma^{-1} \mathbf{1}}$$

Because the sum of weights of any portfolio, $\mathbf{1}' \bar{\omega}$, has to be equal to one, it is easy to show that covariance of any portfolio with the GMV is just equal to the variance of the GMV,

$$\sigma_{GMV,p} = \bar{\omega}'_{GMV} \Sigma \bar{\omega}_p = \frac{\mathbf{1}' \Sigma^{-1} \Sigma \bar{\omega}_p}{\mathbf{1}' \Sigma^{-1} \mathbf{1}} = \frac{1}{\mathbf{1}' \Sigma^{-1} \mathbf{1}} = \sigma_{GMV}^2$$

Therefore, the beta of any asset or portfolio with respect to the GMV is equal to one. This property has two interesting implications. First, together with Equation (A-4), it implies that the marginal contributions to risk of all assets in the GMV are identical, which is equal to the volatility of the GMV. That is,

$$MCTR_{GMV} = \sigma_{GMV} \mathbf{1}$$

Second, together with Equation (A-5), it implies that the vector of percentage contribution to risk is identical to the vector of portfolio weights. That is,

$$PCTR_{GMV} = \bar{\omega}_{GMV}$$

In the general cases where we constrain the portfolio to be fully invested with no cash, the optimal portfolio can be derived by solving

$$\begin{aligned} \max \quad & \bar{\omega}' \mu - \frac{1}{2\lambda} \bar{\omega}' \Sigma \bar{\omega} \\ \text{subject to} \quad & \bar{\omega}' \mathbf{1} = 1 \end{aligned}$$

and the solution is

$$\bar{\omega}_{MVO,Constrained} = \frac{\Sigma^{-1} \mathbf{1}}{\mathbf{1}' \Sigma^{-1} \mathbf{1}} + \lambda \Sigma^{-1} \left[\mu - \mathbf{1} \frac{\mathbf{1}' \Sigma^{-1} \mu}{\mathbf{1}' \Sigma^{-1} \mathbf{1}} \right] \quad (\text{A-8})$$

Substituting Equation (A-7) and noting that $\bar{\omega}'_{GMV} \mu = \mu_{GMV}$, which is the expected excess return of the GMV portfolio, the constrained optimal portfolio can be represented as

$$\bar{\omega}_{MVO,Constrained} = \bar{\omega}_{GMV} + \lambda \Sigma^{-1} [\mu - \mathbf{1} \mu_{GMV}] \quad (\text{A-9})$$

Equation (A-9) suggests that the optimal portfolio has two components, namely, the GMV for minimum variance, and a return enhancement component as partly determined by comparing the expected excess return of each asset to the expected excess return of the GMV. In the special case when expected excess returns of all assets are identical, the second

component is zero, and therefore the investor should hold the GMV as optimal.

Case 3: Most Diversified Portfolio (MDP)

Suppose an active investment manager models expected excess returns of all assets as a constant multiple of their volatilities; that is,

$$\boldsymbol{\mu} = k\mathbf{V} \quad (\text{A-10})$$

where k is a constant multiplier. The resulting optimal portfolio of this active strategy can thus be determined by substituting Equations (A-2) and (A-10) with Equation (A-3), as follows:

$$\boldsymbol{\omega}_{MVO} = \lambda \boldsymbol{\Omega}^{-1} \mathbf{C}^{-1} \boldsymbol{\Omega}^{-1} k \mathbf{V}$$

Define $K = \lambda k$, and since

$$\boldsymbol{\Omega}^{-1} \mathbf{V} = \mathbf{1}$$

in this special case, we get

$$\boldsymbol{\omega}_{MVO} = \boldsymbol{\omega}_{MDP} = K \boldsymbol{\Omega}^{-1} \mathbf{C}^{-1} \mathbf{1} \quad (\text{A-11})$$

which is what Choueifaty and Coignard [2008] called the most diversified portfolio (MDP). In other words, within the context of MVO, the MDP is the optimal portfolio of an active investment strategy that expects Sharpe ratios of all assets to be identical so that expected excess returns are simply the same multiples of their volatilities. The fact that the expected excess returns vector, $\boldsymbol{\mu}$, disappears in the optimal portfolio weights (a result of expected excess returns modeled as a multiple of volatilities, as in Equation (A-10)) as determined by Equation (A-11), makes it qualified to be considered a risk-based strategy.

Matrix algebra shows that $\boldsymbol{\sigma}_{MDP} = K \sqrt{\mathbf{1}' \mathbf{C}^{-1} \mathbf{1}}$. Together with Equations (A-4) and (A-5), it can be shown that

$$PCTR_{MDP} = \boldsymbol{\Omega}^{-1} \mathbf{C}^{-1} \mathbf{1} \cdot \frac{\boldsymbol{\Omega} \mathbf{1}}{\mathbf{1}' \mathbf{C}^{-1} \mathbf{1}}$$

Case 4: Risk Parity Portfolio (RP)

A portfolio is said to be a risk parity portfolio when the percentage contribution to risk of all assets is equal. That is, an RP portfolio must satisfy

$$\boldsymbol{\omega}_i \beta_i = \boldsymbol{\omega}_j \beta_j = \frac{1}{N} \quad (\text{A-12})$$

Case 5: Equally Weighted Brownian Motions Portfolio (EWBM)

In a very interesting and mathematically more rigorous study, Lindberg [2009] derived the solution to mean-variance optimization in continuous time when expected returns of assets are determined by their exposures to a set of unobservable, independent Brownian motions with the same drift. In this particular world, the covariance matrix—not expected returns—is required to derive the optimal portfolio. The optimal portfolio in this case does not equally weight the assets, but rather the Brownian motions that collectively drive return characteristics including volatilities and correlations of assets. As long as assets have different exposures to these Brownian motions, the optimal portfolio is not an equally weighted portfolio of assets.

Equally weighting the Brownian motions, in a sense, is to equal weight risks. The theory and implications appear to be interesting and exciting. However, this approach has one drawback, namely, the decomposition of the covariance matrix into the product of two identical matrices called the volatility matrix, which is not unique. As a result, the choice of which decomposition of the covariance matrix we use—which in turn drives the expected returns and risks of assets—reveals our views on expected returns, after all. For example, in applying the approach to 47 value-weighted industry sector portfolios, Lindberg picks the matrix square root to the covariance matrix to get the volatility matrix required in determining the optimal portfolio.

Although the empirical results are very encouraging in showing a higher Sharpe ratio of the optimal mean-variance efficient portfolio as determined by the methodology versus an equally weighted portfolio and also all the industry sector portfolios, the choice of the matrix square root makes the results only one special case and, therefore, cannot be generalized to support the notion that this approach leads to better investment performance. For example, the choice of a matrix square root implies that the first industry is exposed to the first Brownian motion but not others, the second industry is exposed to the first and second Brownian motions but not others, and the like. Clearly, different orders of industries in this case can lead to different results and, therefore, the results are ultimately dependent on our implicit views of expected returns through the choice of the volatility matrix. As a matter of fact, Lindberg [2009] illustrated how to incorporate expected returns in his framework: “[T]he investor can design freely a target matrix that, when considered as a volatility matrix, has row sums that lead to continuously compounded rates of return that reflect the investor’s market views” (p. 468).

ENDNOTES

I am grateful for the helpful discussions and debates with George Chacko, Hakan Kaya, and Bobby Pornrojngkool on the subject of this article. I would like to thank many colleagues at Neuberger Berman for their valuable comments, especially Alex Da Silva, Alan Dorsey, and members of the Quantitative Investment Group. Reviews and feedback by Sidney Chong; Hua Fan; André Perold; participants in the 3rd Annual Australia Investment Management Summit 2010; the Institute of Fiduciary Education: Market Makers–2010 Seminar; the Teacher Retirement System of Texas Strategic Partnership Network Summit–July 2010; the CFA Society of San Francisco’s Continuing Education Program on December 9, 2010; the CFA Society of San Diego’s Continuing Education Program on March 16, 2011; and the Chicago Quantitative Alliance Spring 2011 conference are much appreciated. This article reflects the views of the author and does not reflect the official views of the author’s employer, Neuberger Berman.

¹In this article, 60/40 denotes a portfolio with 60% in stocks and 40% in bonds. Whereas there are many combinations, we follow the pundits in using 60/40 to generalize the typical optimal strategic portfolio for the ease of reference in this article.

²See Liesching [2010], for example.

³A portfolio is said to be more efficient if it delivers higher return for a given level of risk, a lower risk for a given level of return, or both. Risk can be defined in multiple dimensions but, in this article, unless stated otherwise, risk is defined as volatility of returns. One should distinguish the difference between portfolio efficiency, as defined earlier, versus the general efficient market hypothesis on the informational efficiency of how assets are priced. Our discussions in this article focus on portfolio efficiency.

⁴The Grossman and Stiglitz paradox points out the fact that in a world where information discovery is not free, and if the market is informational efficient so that prices reflect all information available, then investors would stop their research effort and, instead, infer valuable information from prices. In this scenario, however, there must be valuable information that is missed by prices and, therefore, the paradox exists. As a result, the market is informational efficient only to the point such that the marginal benefit of additional information is the same as the marginal cost of information discovery.

⁵We will argue in subsequent sections that not using expected excess returns as inputs does not necessarily mean that returns are not being forecasted.

⁶Constraints, however, may rule out an analytical solution and the problem has to be solved numerically.

⁷See Da Silva, Lee, and Pornrojngkool [2008] for graphical illustration and numerical examples in Excel.

⁸For example, many pension plans are said to have an actuarial return assumption between 7.5% and 8.0%. When this required return is input as a constraint in the optimization process, the optimal portfolio can invest disproportionately more into risky assets, such as stocks, in order to achieve a certain expected return requirement and, therefore, the concentration of risk contribution from stocks, if any, is *intentional* in this case.

⁹A large amount of literature on the relation between risk and expected return, both cross-sectional and intertemporal, can be found. The class of GARCH-in-mean studies, for example, models expected return as a linear function of expected variance. In a more recent study, Martellini [2008] used volatilities as proxies for expected returns in an attempt to design an efficient equity benchmark.

¹⁰As discussed in Case 3, one of the constraints that Choueifaty and Coignard [2008] imposed in constructing the MDP is to limit contribution to risk to 4% per asset.

¹¹Implied expected excess return analysis cannot pin down the absolute magnitude of the returns but instead requires a scale parameter or a predetermined expected excess return of one particular asset as the anchor point for the others. In our analysis, we use a scale parameter so that the resulting Sharpe ratio of the market capitalization-weighted portfolio is 0.5.

¹²As a related topic, there have been numerous discussions in the asset management industry on whether it makes sense to measure portfolio managers’ performance against a benchmark. For instance, some equity portfolio managers claim to be benchmark-agnostic managers in an attempt to produce absolute returns, rather than simply outperforming a particular assigned benchmark. From an investor’s opportunity cost perspective, however, the easiest thing an investor can do to guarantee better-than-average performance net of management fees is to invest in the market-capitalization benchmark. To that end, we think benchmarking portfolio managers’ performance against a market-capitalization portfolio still makes sense.

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